THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
"Igor Sikorsky Kyiv Polytechnic Institute"

Department of Mathematical Physics and Differential Equations

HIGHER MATHEMATICS PART 2. Differential and Integral Calculus of the Function of Several Variables. Differential Equations.

SYLLABUS

Description of the Discipline	
Level of HE	*The first bachelor*
Branch of Knowledge	*13 Mechanical engineering*
Specialty	*131 "Applied Mechanics"*
Educational Program	*Manufacturing Engineering*
Status of the Discipline	*Compulsory.*
Form of Study	*Full-time*
Year and Semester	*First year, spring semester*
Number of Credits	*8,5 credits ECTS (255 hours), including 72 hours of Lectures, 72 hours pf Practical Lessons, 111 hours of Self-study work*
Semester's Control	*Exam/ Midterm test, Individual tasks*
Schedule	*According to the schedule on the university website*
Language	*English*
Information About Course Teacher	*Ganna Zhuravska, Ph.D., Associate Professor* https://mph.kpi.ua/osobovij-sklad.html
Learning Resources	*Learning resources are determined by the course teacher and are given to students in on the first lesson. They consist of links to remote resources in Moodle, Google classroom, information resources in the university library, on the website of the department, etc.*

Course Description

1. Learning Objectives, Tasks and Outcomes of the Discipline

The discipline "Higher Mathematics Part 2. Differential and Integral Calculus of the Function of Several Variables. Differential Equations" is the second credit module belongs to the discipline "Higher Mathematics", which is included in the cycle of natural-scientific trainings for bachelors of the specialty 131 "Applied Mechanics".

The purpose of the discipline is studying the basic concepts and methods of integral calculus of the function of one variable, differential and integral calculus of the function of several variables, vector calculus, ordinary differential equations; developing of the student's capability to use theory for solving typical problems on these topics; application of acquired knowledge, skills and abilities to solve applied problems of mathematics, mechanics, physics and in their daily practice; independent usage and studying of mathematical literature and other information sources.

Objective of the discipline is shaping the following knowledge and skills of students.

According to the matrix of correspondence of program competencies

- *general competencies:*

GC1. Ability to abstract thinking, analysis and synthesis;

- *professional competencies:*

PC1. Ability to analyze materials, structures and processes based on laws, theories and methods of mathematics, natural sciences and applied mechanics.

Students must demonstrate the following learning outcomes.

According to the matrix of correspondence of program learning outcomes in the educational program

- LO1) Ability to choose and apply convenient mathematical methods for solving problems of applied mechanics.
- **Knowledge:** the bases of the integral calculus of functions of one and many variables (indefinite integral, properties of an indefinite integral, a table of integrals, methods of integration); the bases of the integral calculus of functions of several variables (definite integral and its geometric interpretation, fundamental theorem of calculus, improper integrals, double and triple integrals, methods of calculation in different coordinate systems, line and surface integrals, application of all kinds of integrals to problems of geometry and physics, vector calculus); the fundamentals of the theory and practice of ordinary differential equations (differential equations of the first order, separable, homogeneous and Bernoulli differential equations, the initial value problem, differential equations of higher orders, linear constant coefficients homogeneous and non homogeneous differential equations of higher orders, reduction of order).
- **Skills:** to find indefinite integrals by basic methods of integration; to find indefinite integrals of rational, fractional-rational, trigonometric, irrational functions by basic methods of integration; to calculate of definite integrals, to apply integrals; to find partial derivatives and differentials for a function of several variables; to apply partial derivatives; to find extrema of a function of two variables; to find the gradient and directional derivative of a scalar field; double, triple, line and surface integrals in different systems of coordinates, to apply all kinds of integrals; to find general and partial solutions of differential equations; to find general and partial solutions of systems of linear constant coefficients equations.
- **Experience:** thinking logically and flexibly; independently using and studying literature on mathematics; generalizing, setting the goal and choosing ways to solve it; using methods of mathematics in engineering calculations.

2. Prerequisites and Postrequisites of the Discipline (place in the structural and logical scheme of education according to the relevant educational program)

Interdisciplinary connections:

The discipline "Higher Mathematics Part 2. Differential and Integral Calculus of the Function of Several Variables. Differential Equations" is closely related to the disciplines "Higher Mathematics Part 3. Series. Theory of the function of a complex variable", which are studied in the following semesters. This discipline provides other disciplines of the natural sciences cycle such as Physics, Engineering Graphics, Computer Science, Electrical Engineering and Electronics; must precede and provide the following disciplines in the program of professional and practical training of a specialist: Engineering Mechanics, *Strength of Materials*, Technology of Structural Materials, Theory of Mechanisms and Machines, Theoretical Mechanics, Mathematical Modeling and other disciplines (according to the structural and logical scheme of the educational program).

3. Content of the Discipline

Part 3. Integral Calculus of Functions of One Variable (continuation).

Topic 3.3. Integration of Rational, Irrational and Trigonometric Functions.

Topic 3.4. The Definite Integral. Properties of the Definite Integral. Fundamental Theorem of Calculus (Newton-Leibniz Formula). Techniques of Evaluating Definite Integrals (Integration by Parts; Integration by Substitution). Improper Integrals.

Topic 3.5. Application of the Definite Integral. The Area of a region. The Volume of a Solid of Revolution. The Arc length. The Surface of a Solid of Revolution.

Part 4. Differential Calculus of Functions of Several Variables.

Topic 4.1. The Concept of a Function of Several Variables. Limit and Continuity of a Function of Several Variables.

Topic 4.2. Partial Derivatives of a Function of Several Variables. Partial Derivatives of Higher Orders. The Partial Derivatives of a Composite Function.

Topic 4.3 Differentials of a Function of Several Variables and Their Properties. The Partial Derivatives of an Implicit Function.

Topic 4.4. The Tangent Plane and the Normal Line to a Surface. Taylor's Formula for the Function of Two Variables. Local Extrema of a Function of Two Variables. The Largest and Smallest Value of a Function of Two Variables in the Region.

Part 5. Multiple Integrals.

Topic 5.1. The Concept of a Double Integral. Properties. Calculating Double Integral in Cartesian Coordinates.

Topic 5.2. Changing Variables in a Double Integral. Double Integral in Polar Coordinates. Application of a Double Integral to Problems of Geometry and Mechanics.

Topic 5.3. The Concept of a Triple Integral. Properties. Calculating Triple Integral in Cartesian Coordinates.

Topic 5.4. Changing Variables in a Triple Integral. Calculating Triple Integral in Cylindrical and Spherical Coordinates. Application of the Triple Integral to Problems of Geometry and Mechanics.

Part 6. Line Integrals.

Topic 6.1. The Concept of Line Integrals with Respect to Arc Length. Properties, Calculating and Application.

Topic 6.2. The Concept of Line Integrals of Vector Fields. Properties, Calculating and Application.

Topic 6.3. Green's formula. Independence of Path.

Part 7. Surface Integrals.

Topic 7.1. The Concept of Surface Integrals. Properties. Oriented Surfaces.

Topic 7.2. Calculating Surface Integrals. Application.

Topic 7.3. Ostrogradsky's Formula. Stokes' formula.

Part 8. Vector calculus.

Topic 8.1. Basic Concepts of Vector calculus. Scalar Field and Its Properties. Gradient. Directional Derivative.

Topic 8.2. Vector field. Flux of a Vector Field. Divergence. Divergence Theorem.

Topic 8.3. Circulation around the closed curve. Curl.

Topic 8.4. The Hamiltonian Operator and Certain Applications of It.

Part 9. First-Order Differential Equations.

Topic 9.1. Basic Concepts and Definitions. The Initial Value Problem. Differential Equations with Separable Variables. Homogeneous Differential Equations. Exact Differential Equations.

Topic 9.2. First-Order Linear Differential Equations. Bernoulli's equation.

Part 10. Higher-Order Differential Equations.

Topic 10.1. Basic Concepts and Definitions. Some Types of Second-Order Differential Equations Reducible to First-Order Equations.

Topic 10.2. Second-Order Homogeneous Linear Differential Equations. Functions that Are Linearly Independent on an Interval. Wronskian. Fundamental System of Solutions.

Topic 10.3. Second-Order Homogeneous Linear Differential Equations with Constant Coefficients. Characteristic Equation. General solution.

Topic 10.4. Second-Order Nonhomogeneous Linear Differential Equations. The Method of Variation of Arbitrary Constants.

Topic 10.5. Second-Order Nonhomogeneous Linear Differential Equations with Constant Coefficients and Special Right-Hand Side.

Topic 10.6. Systems of Differential Equations: Basic Concepts and Definitions. Systems of Linear Differential Equations with Constant Coefficients.

4. Informational Support

Readings

1. H. Jerome Keisler Elementary Calculus: an Infinitesimal Approach/ H. Jerome Keisler – On-line Edition. 2000 https://www.math.wisc.edu/~keisler/calc.html
2. Higher Mathematics. Multivariable Calculus. Vector Calculus. Elements of Theory [Electronic resource] / Igor Sikorsky Kyiv Polytechnic Institute ; compiler: Ganna Zhuravska. – Electronic text data (1 file: 3,13 MB). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2021. – 110 p. https://ela.kpi.ua/handle/123456789/41246
3. Higher Mathematics. Integral Calculus of a Function of One Variable. Elements of Theory [Electronic resource] / Igor Sikorsky Kyiv Polytechnic Institute ; comp. G. V. Zhuravska. – Electronic text data (1 file: 1,31 Mb). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2019. – 68 p. https://ela.kpi.ua/handle/123456789/27854
4. Swokowski, Earl William Calculus: 5th ed / Swokowski, Earl William - Published 1991 by Pws-Kent Publishing Company – ISBN 0-534-92492-1 - 1053 p.
5. Jeffrey R. Chasnov Introduction to Differential Equations. Lecture notes for MATH 2351/2352 / Jeffrey R. Chasnov – The Hong Kong University of Science and Technology, 2016 – 147p. http://www.math.ust.hk/~machas/differential-equations.pdf

Educational Content

5. Learning Activities and Teaching Methods

Basic teaching methods:

1. Providing during lectures theoretical material and examples of solving the main practical problems.

2. Solving examples and tasks during practical trainings through active and collective learning, part-search and research methods;

3. Self-studying of students, such as homework to each topic of the course and individual tasks.

4. As an addition to traditional training activities there are proposed such activities as internet mailing, distance and online learning, online testing etc.

5. Assessment is carried out with the help of various control measures: midterm tests, mathematical dictation, classroom activities, homework, individual tasks and final exam.

Lectures

№	The title of the lecture and a list of the main subtopics
1	Basic methods of Integration: Integration by Substitution, Integration by Parts.
2-3	Integration of Rational Functions.

4	Integration of Irrational Functions.
5	Integration of Irrational Functions. Task for the Self-study: Euler's Substitutions
6	Integration of Trigonometric Functions.
7	The Definite Integral. Properties of the Definite Integral. Fundamental Theorem of Calculus (Newton-Leibniz Formula). Techniques of Evaluating Definite Integrals (Integration by Parts; Integration by Substitution).
8	Improper Integrals.
9-10	Application of the Definite Integral. The Area of a region. The Volume of a Solid of Revolution. The Arc length. The Surface of a Solid of Revolution.
11	The Concept of a Function of Several Variables. Limit and Continuity of a Function of Several Variables. Partial Derivatives of a Function of Several Variables.
12	Partial Derivatives of Higher Orders. Differentials of a Function of Several Variables and Their Properties.
13	The Partial Derivatives of an Implicit Function. The Partial Derivatives of a Composite Function. The Tangent Plane and the Normal Line to a Surface.
14	Local Extrema of a Function of Two Variables. The Largest and Smallest Value of a Function of Two Variables in the Region.
15	The Concept of a Double Integral. Properties. Calculating Double Integral in Cartesian Coordinates.
16	Changing Variables in a Double Integral. Double Integral in Polar Coordinates. Application of a Double Integral to Problems of Geometry and Mechanics.
17	The Concept of a Triple Integral. Properties. Calculating Triple Integral in Cartesian Coordinates.
18	Changing Variables in a Triple Integral. Calculating Triple Integral in Cylindrical and Spherical Coordinates. Application of the Triple Integral to Problems of Geometry and Mechanics.
19	The Concept of Line Integrals with Respect to Arc Length. Properties, Calculating and Application.
20	The Concept of Line Integrals of Vector Fields. Properties, Calculating and Application.
21	Green's formula. Independence of Path.
22	The Concept of Surface Integrals. Properties. Oriented Surfaces.
23	Calculating Surface Integrals. Application.
24	Ostrogradsky's Formula. Stokes' formula.
25	Basic Concepts of Vector calculus. Scalar Field and Its Properties. Gradient. Directional Derivative.
26	Vector field. Flux of a Vector Field. Divergence. Divergence Theorem.

27	Circulation around the closed curve. Curl.
28	The Hamiltonian Operator and Certain Applications of It.
29	Basic Concepts and Definitions. The Initial Value Problem. Differential Equations with Separable Variables. Homogeneous Differential Equations. Exact Differential Equations.
30	First-Order Linear Differential Equations. Bernoulli's equation.
31	Basic Concepts and Definitions. Some Types of Second-Order Differential Equations Reducible to First-Order Equations.
32	Second-Order Homogeneous Linear Differential Equations. Functions that Are Linearly Independent on an Interval. Wronskian. Fundamental System of Solutions.
33	Second-Order Homogeneous Linear Differential Equations with Constant Coefficients. Characteristic Equation. General solution.
34	Second-Order Nonhomogeneous Linear Differential Equations. The Method of Variation of Arbitrary Constants.
35	Second-Order Nonhomogeneous Linear Differential Equations with Constant Coefficients and Special Right-Hand Side.
36	Systems of Differential Equations: Basic Concepts and Definitions. Systems of Linear Differential Equations with Constant Coefficients.

Practical Trainings

№	The title of the practical training
1	Basic methods of Integration: Integration by Substitution, Integration by Parts.
2-3	Integration of Rational Functions.
4-5	Integration of Irrational Functions.
6	Integration of Trigonometric Functions.
7	The Definite Integral. Fundamental Theorem of Calculus (Newton-Leibniz Formula). Techniques of Evaluating Definite Integrals.
8-9	Improper Integrals.
10	Application of the Definite Integral. The Area of a region. The Volume of a Solid of Revolution. The Arc length. The Surface of a Solid of Revolution.
11	Midterm Test 1.
12	The Concept of a Function of Several Variables. Limit and Continuity of a Function of Several Variables. Partial Derivatives of a Function of Several Variables.
13	Partial Derivatives of Higher Orders. The Partial Derivatives of a Composite Function.
14	Differentials of a Function of Several Variables. The Tangent Plane and the Normal Line to a Surface.

15	Local Extrema of a Function of Two Variables. The Largest and Smallest Value of a Function of Two Variables in the Region. (1 hour) Midterm Test 2. (1 hour)
16	Double Integral. Calculating Double Integral in Cartesian Coordinates.
17	Changing Variables in a Double Integral. Double Integral in Polar Coordinates.
18	Application of a Double Integral to Problems of Geometry and Mechanics.
19	Triple Integral. Calculating Triple Integral in Cartesian Coordinates.
20	Changing Variables in a Triple Integral. Calculating Triple Integral in Cylindrical and Spherical Coordinates.
21	Application of the Triple Integral to Problems of Geometry and Mechanics. (1 hour) Midterm Test 3. (1 hour)
22	Line Integrals. Calculating and Application.
23	Green's formula. Independence of Path.
24	Surface Integrals. Calculating and Application.
25	Ostrogradsky's Formula. Stokes' formula.
26	Basic Concepts of Vector calculus. Scalar Field and Its Properties. Gradient. Directional Derivative.
27	Vector field. Flux of a Vector Field. Divergence. Divergence Theorem. Curl.
28	Circulation around the closed curve. (1 hour) Midterm Test 4. (1 hour)
29	Differential Equations with Separable Variables. Homogeneous Differential Equations. The Initial Value Problem.
30	First-Order Linear Differential Equations. Bernoulli's equation
31	Second-Order Differential Equations Reducible to First-Order Equations.
32	Second-Order Homogeneous Linear Differential Equations. Functions that Are Linearly Independent on an Interval. Wronskian. Fundamental System of Solutions. Second-Order Homogeneous Linear Differential Equations with Constant Coefficients. Characteristic Equation. General solution.
33	Second-Order Nonhomogeneous Linear Differential Equations with Constant Coefficients and Special Right-Hand Side.
34	Second-Order Nonhomogeneous Linear Differential Equations. The Method of Variation of Arbitrary Constants.
35	Systems of Linear Differential Equations with Constant Coefficients.
36	Midterm Test 5.

6. Self-studying of Students

There are 111 hours of self-study work of students. They include: 61 hours for learning lectures and doing homework, 10 hours for preparing to midterm tests, 10 hours for doing individual tasks and 30 hours for preparing for final exam.

Self-study work also carries out studying some topics of the discipline.

Self-study Work

№	The title of topic for the self-study	Hours
Part 3. Integral Calculus of Functions of One Variable		
1	Euler's Substitutions	2
Part 4. Differential Calculus of Functions of Several Variables.		
2	Taylor's Formula for the Function of Two Variables. Conditional Extrema.	2

Policies and Assessments

7. Policies of the Discipline

Class-meetings are held in auditoriums according to the schedule. In the case of online classes that should be provided by the relevant order of the university, lectures and practical trainings are engaged in online video communications (Zoom, Google-Meet, Skype).

Requirements and the system of assessment are announced to students in the first lesson.

Attendance

Students are expected to attend each class meeting, since during these lessons they study theoretical material and develop the skills needed to complete homeworks, individual tasks and middle tests. There are no penalty points for absence from lectures and practical classes.

Absence During Control

Failure to submit the scheduled time the individual tasks and systematic failure to do homeworks without a valid reason is punishable by penalty points, according to the Rating System. Absence from the middle test, if the reason is not documented, is not give you a chance to do it in another time.

Academic Integrity Policy

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

Norms of Ethical Behavior

Norms of ethical behavior of students and teachers are defined in Section 2 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".
Read more: https://kpi.ua/code.

8. Assessment of Learning Outcomes and Rating System

Execution ongoing control and assessment of student's knowledge are made due to classroom activities, middle tests and individual tasks, the purpose of which is to identify the level of assimilation of relevant topics.

The final test is an exam in the end of each semester. The 100-points rating system and the university scale are used to evaluate the results of the training.

The Rating System

1. The student rating of the credit module is calculated from 100 points, 60 of them are the starting scores. Starting scores (during the semester) consists of the points that the student receives for:

1) classroom activities (36 lessons);
2) homework;
3) five midterm tests;
4) individual tasks.

2. Criteria For Scoring Points

2.1. Classroom Activities:

– student has a thorough knowledge and understanding of the content and a high level of competence in the processes and skills - 1 point;

– student has achieved a basic level of competence in the processes and skills - 0.5 points;

– student has no elementary knowledge and understanding of the content - 0 points.

The maximum of points for classroom activities during the semester is 18.

2.2. Homework

It includes completing tasks at home for each topic covered. Correctly completed and timely homework is assessed at 0.2 points.

The maximum of points for all homework is 0.2 ×30=6 points.

2.3. Midterm Test

There are four Midterm tests (two for 6 points and three for 4 points)

- complete solutions (no less than 90% of the required information) – 5.5-6 points or 3.5-4 points;
- solutions with minor inaccuracies (no less than 75% of the required information) - 4.5-5 points or 3 points;
- mistakes in an solutions (no less than 60% of the required information) - 3,5-4 points or 2.5 points;
- no solutions - 0-3 points or 0-2 points.

Midterm test №1	
Topic: "Integral Calculus of a Function of One Variable" (2 hours)	Max points 6
Midterm test №2	
Topic: "Multiple Integrals" (1 hours)	Max points 4
Midterm test №3	
Topic: "Vector Calculus" (1 hours)	Max points 4
Midterm test №4	
Topic: "Differential Equations" (2 hours)	Max points 8

Use of a cell phone or any other communication device is not permitted during midterm tests. Violation of this policy will result in a score of 0 (zero) in the midterm tests.

Make-ups of the midterm test is allowed only in the case of a valid reason.

In the case of online learning, the midterm test is written by students in practical classes using Zoom or Skype platforms (or another, depending on the agreement with the teacher).

Students are sent the card for the test, and they, in two academic hours, must send photocopies of written solution via messenger (Telegram, What's Up etc.) or by e-mail. If the student's work is not sent in time, it is considered that the student was absent from the test, the work is not checked, and student receives 0 points.

2.4. Individual Tasks - 16 points.

- complete solutions – 14.5-16 points;
- solutions with minor inaccuracies - 12-14 points;
- mistakes in an solutions - 9.5-11.5 points;
- no solutions - 0-9 points.

Each week of delay of the submission of the individual tasks to examination are provided with penalty (-1) points.

The student must submit the completed individual tasks no later than the last lesson of the semester. In case of violation of this deadline, the student is considered not admitted to the exam of the main session. Student could be allowed to pass exam should submit the individual tasks before the additional session.

In the case of online learning, the individual tasks is checked by sending photocopies of the written work to the teacher's e-mail (or other platform, depending on the agreement with the teacher (Telegram, What's Up etc.)).

3. The requirement to the first midterm assessment (attestation) is to receive at least 11 points and to submit the corresponding part of individual tasks. The requirement to the second midterm assessment (attestation) is to receive at least 22 points and to submit the corresponding part of individual tasks.

4. The allowance requirement to passing the exam is the individual tasks, that must be credited, and no less than 36 points of starting score.

5. At the exam each student have two theoretical questions (8 points each) and three practical ones (8 points each). Total exam score is 40 points.

The criterion for evaluating theoretical questions:

- full answer (not less than 90% of the required information) – 7.5-8 points.
- sufficiently complete answer (not less than 75% of the required information, or minor inaccuracies) - 6-7 points.
- incomplete answer (not less than 60% of the required information and some mistakes) - 4-5 points.
- wrong answer – 0-4 points.

Criteria for evaluating practical tasks:

- complete solution of the task - 8 points.
- solution of the task with minor inaccuracies - 6-7 points.
- the task is solved with some mistakes - 4-5 points.
- the task is not solved – 0-4 points.

If the exam score is less than 24 points, then the student gets an "unsatisfactory" rating.

Use of books, a cell phone or any other communication device is not permitted during exams. Violation of this policy will result in a score of 0 (zero) in the exam.

6. The sum of the starting score and the exam score (personal score) is transferred into the grade of credit module according to the table:

PERSONAL SCORE	GRADE
95-100	Excellent
85-94	Very Good
75-84	Good
65-74	Satisfactory
60-64	Sufficiently
Personal score<60 or the exam score is less than 24 points	Unsatisfactory
Personal score < 36 or the individual tasks is not credited	Not allowed to pass exam

In the case of online learning, according to the teacher's decision, it is possible to set the exam grade by recalculating the starting points on a 100-point scale:

$$R = 60 + \frac{40(R_I - R_D)}{R_C - R_D},$$

where R_I - the amount of points earned by the student during the semester (starting score).

If the student does not agree with that grade, he takes the exam in video mode according to the schedule of the examination session.

Syllabus of the Discipline:

Compiled:

Inna Kopas, Ph.D., Associate Professor

Tamara Karpalyuk, Ph.D., Associate Professor

Natalia Stepanenko, Ph.D., Associate Professor

Translated:

Ganna Zhuravska, Ph.D., Associate Professor

Approved:

Department of Mathematical Physics and Differential Equations (Protocol №1 dated July 22, 2023)

Agreed:

Methodical Commission of Institute of Mechanical Engineering (Protocol №9 dated July 30, 2023)